



SPECIFICATION FOR FINISHED 1.44-METER PRIMARY MIRROR SEGMENTS

TMT.OPT.SPE.07.002.CCR06

April 15, 2019



SIGNATURE PAGE

Author Release Note:

See Document Change Record for change requests incorporated at this time.

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DOCUMENT CHANGE RECORD

Revision	Change Approval	Release Approval	Date Released
CCR06	CR307: Collection-24217	Docushare Routing #48274	April 15, 2019
CCR05	CR274: Collection-20024 ADMIN Changes	Docushare Routing #35129	June 29, 2018
CCR04	Initial CCR release under Configuration Management	Docushare Routing #3777	19 February 2016
CCR03	NOTE: This revision was never formally released (Pre-release)		APR 2008
DRF02	Updated Draft		25 Mar, 2008
REL01	Initial Release		7 Mar, 2008

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1. INTRODUCTION

1.1 PURPOSE

The TMT primary mirror is a 30-meter aperture composed of 492 hexagonal Segments. The out-of-plane positions and orientations of the Segments will be controlled continuously so that the array functions as a single mirror. The primary mirror is divided into six identical sectors, resulting in six-fold symmetry in the array and 6 instances of each of 82 unique Segment Types. Therefore, 82 unique optical surface configurations are required for Polished Mirror Assemblies in their final form.

The finished Segments will be hexagons approximately 1.44 meters as measured across corners. However, the Segments are not regular hexagons and they are not identical. The finished segments will be approximately-uniform-thickness optics, except for variations caused by optical surface asphericity. Each of the 82 Segment Types has slightly different dimensions, within a range of a few millimeters, and each has a different best-fit radius of curvature of the Optical Surface.

Each Segment will be mounted on a Segment Support Assembly (SSA). Each SSA is tuned for a specific Type of segment using different whiffletree-mounted counterweights, so there will be 82 types of SSAs. The finished Segment mounted onto its SSA, as shown in (AD6), is termed a Polished Mirror Assembly. A Primary Segment Assembly (PSA) is the combination of a Polished Mirror Assembly with a Subcell that attaches it to the telescope, plus the M1 Control System actuators and sensors.

1.2 SCOPE

This document specifies the requirements for the finished Primary Mirror Segments, as mounted in Polished Mirror Assemblies (PMAs). Each PMA consists of a polished hexagonal segment mounted on a Segment Support Assembly (SSA), as depicted in the TMT M1 Polished Mirror Assembly drawing, (AD6).

This specification and the following Documents completely define the mechanical and optical characteristics of the 82 Types of PMAs:

- (1) TMT M1 Polished Mirror Assembly, (AD6)
- (2) TMT PMA Intermediate Polishing Specification, (AD7)
- (3) MIL-PRF-13830B (RD2)

The conventions and instructions for utilizing the M1S Segmentation Database (AD10) are described in Appendix A (Section 5.1).

1.3 APPLICABLE DOCUMENTS

AD1 DELETED

AD2 DELETED

AD3 DELETED

AD4 DELETED

AD5 DELETED

AD6 TMT M1 Finished Polished Mirror Assembly

CAD Drawing No: M1S-001-11000 Rev D

TMT.OPT.DWG.07.002 REL06

<https://docushare.tmt.org/docushare/dsweb/Get/Version-59957>

AD7 TMT Polished Mirror Assembly - Intermediate Polishing Specification

TMT.OPT.SPE.11.001 CCR06

<https://docushare.tmt.org/docushare/dsweb/Get/Version-95583>

AD8 Dimensioning and Tolerancing

ASME Y14.5 Edition 2009

<https://www.asme.org/products/codes-standards/y145-2009-dimensioning-and-tolerancing>

AD9 M1 Segment Finished Segment Specification - Vendor Table

TMT.SEN.TEC.12.002 REL12

<https://docushare.tmt.org/docushare/dsweb/Get/Version-88754>

AD10 TMT M1S Segmentation Database

TMT.OPT.TEC.07.044

<https://docushare.tmt.org/docushare/dsweb/Get/Version-61423>

1.4 REFERENCE DOCUMENTS

RD1 Specification for Primary Mirror Segment Blanks

TMT.OPT.SPE.07.001

<https://docushare.tmt.org/docushare/dsweb/Get/Document-6266>

RD2 Optical Components for Fire Control Instruments; General Specification Governing the Manufacture, Assembly, and Inspection of

MIL-PRF-13830

<http://quicksearch.dla.mil/Analyse/ImageRedirector.aspx?token=14437.10290>

RD3 TMT M1 Polished Segment Drawing

CAD Drawing No: M1S-001-01000

TMT.OPT.DWG.14.005

<https://docushare.tmt.org/docushare/dsweb/Get/Document-33140>

RD4 DELETED

RD5 TMT M1 Segment Polishing Specification Analysis

TMT.SEN.TEC.11.018

<https://docushare.tmt.org/docushare/dsweb/Get/Document-19870>

RD6 TMT M1 SSA Edge Sensor Simulator

CAD Drawing No: M1S-910-00802

TMT.OPT.DWG.15.011

<https://docushare.tmt.org/docushare/dsweb/Get/Document-51297>

1.5 ABBREVIATIONS

CRS	Coordinate Reference System
EIDP	End Item Data Package
M1	Primary Mirror
M1CS	Primary Mirror Control System
M1S	M1 System
PMA	Polished Mirror Assembly
PSA	Primary Segment Assembly
PSSN	Normalized Point Source Sensitivity
RMS	Root Mean Square
SSA	Segment Support Assembly
TMT	The Thirty Meter Telescope Project or the Thirty Meter Telescope
WFE	Wavefront Error

1.6 DEFINITIONS

Acceptance Tests. The term “Acceptance Tests” is defined in Section 1.8.

Back Surface. The Back Surface is the convex surface of the Segment that is on the opposite side from the Optical Surface, also called S2.

Basic Dimension. A Basic Dimension is a dimension that describes the theoretically perfect size, shape, or location of a feature. Geometric Dimensioning and Tolerancing shall be interpreted per ASME Y14.5-2009 (AD7).

Blank. A glass or glass-ceramic substrate from which a hexagonal mirror Segment will be fabricated. The Blanks are described in the *Specification for Primary Mirror Segment Blanks* (RD1).

Blank Supplier. The company that will fabricate the Blanks.

Chip. The term Chip is defined in Section 2.4.6.

Generating. Machining the surfaces of the Blank by fixed-abrasive grinding.

M1 Coordinate Reference System. The M1 Coordinate Reference System (M1CRS) is defined in Section 2.2.1.

Observatory. The term Observatory refers to the facility on Mauna Kea, Hawaii that will incorporate the Thirty Meter Telescope.

Optical Surface. The Optical Surface is defined in Section 2.4, also called S1.

Polished Mirror Assembly. The term Polished Mirror Assembly is defined in Section 1.1.

Primary Segment Assembly. The term Primary Segment Assembly (PSA) is defined in Section 1.1.

PSA Coordinate Reference System. The PSA Coordinate Reference System (PSACRS) is defined in Section 2.2.2.

Segment. A Segment is one of the hexagonal mirrors that, in combination, form the surface of the TMT primary mirror.

Segment Blank Specification. The Segment Blank Specification is a TMT document (RD1).

Subcell. The part of the PSA that is permanently installed and aligned on the telescope.

Subsurface Damage. Cracks in the glass below the surface caused by any process step such as machining or grinding, whether visible or not.

Supplier. A company or institution that contracts with the TMT Partner to produce Polished and Mounted PMAs.

Type. "Type" (when spelled with a capital T) refers to one of the 82 unique Segment or PMA configurations, for example: "Type-57".

1.7 ACCEPTANCE TESTING

All verification activities are the responsibility of the Contractor; i.e., the Contractor shall be solely responsible for providing any and all test equipment, analyses, inspections, and other means necessary to verify that the specifications and requirements have been met.

Examples of verification methods include:

- **Analysis.** Verification by analysis shall mean that Contractor analytically demonstrates that the design meets the specification. Such analyses may include finite element methods, computation fluid analyses, closed form analyses, etc. All analyses shall be provided to TMT in written report form, in both electronic (e.g., MS Word) and paper copy format and be part of the EIDP.
- **Inspection.** Verification by inspection shall mean that the Contractor visually demonstrates to TMT personnel that the specification has been achieved on the as-built equipment during factory acceptance testing.
- **Acceptance Test or Test.** Verification by test &/or measurement shall mean that Contractor empirically demonstrates that the as-built equipment meets the specification. Testing may be required in the factory during factory acceptance testing and/or at the Delivery Location.

All analyses, test measurements (with test error analysis) and other verification results shall be provided to TMT in written report form, in both electronic (e.g., MS Word or Excel) and paper copy format. For each test method used for acceptance testing, the Contractor shall perform a test error analysis. All potential errors effecting the measurement shall be listed and their influence on the test results evaluated. The required measurement value shall be adjusted so the Test shall confirm that the specification has been met after taking the test error analysis into account.

2. SPECIFIC REQUIREMENTS

2.1 GLOBAL PROPERTIES OF PRIMARY MIRROR

Definition: The TMT primary mirror is a hyperboloid. The expression for a conic surface of revolution is:

$$Z_{M1}(X_{M1}, Y_{M1}) = \{R - [R^2 - (K + 1)r^2]^{1/2}\} / (K + 1)$$

where K is the conic constant, R is the paraxial radius of curvature, and $r = (X_{M1}^2 + Y_{M1}^2)^{1/2}$. X_{M1} , Y_{M1} and Z_{M1} are in the $M1$ Coordinate System, as defined in Section 2.2.1.

[SPE-M1.SEG.POL-1100] For the TMT primary mirror, the conic constant $K = -1.00095348$ and the paraxial radius of curvature $R = 60.0$ meters. The conic constant and the paraxial radius of curvature shall be considered Basic Dimensions.

2.2 COORDINATE SYSTEMS

2.2.1 M1 Coordinate System

Definition: The global coordinate system for the TMT primary mirror is shown in Figure 2-1. Coordinates in this system are designated by the subscript $M1$, for example: X_{M1} . This $M1$ Coordinate System is a right-handed system. The Z_{M1} -axis is the optical axis of the telescope, positive towards the sky, and the X_{M1} -axis is parallel to the telescope elevation axis. The positive Y_{M1} -axis points to the sky when the optical axis points to the horizon.

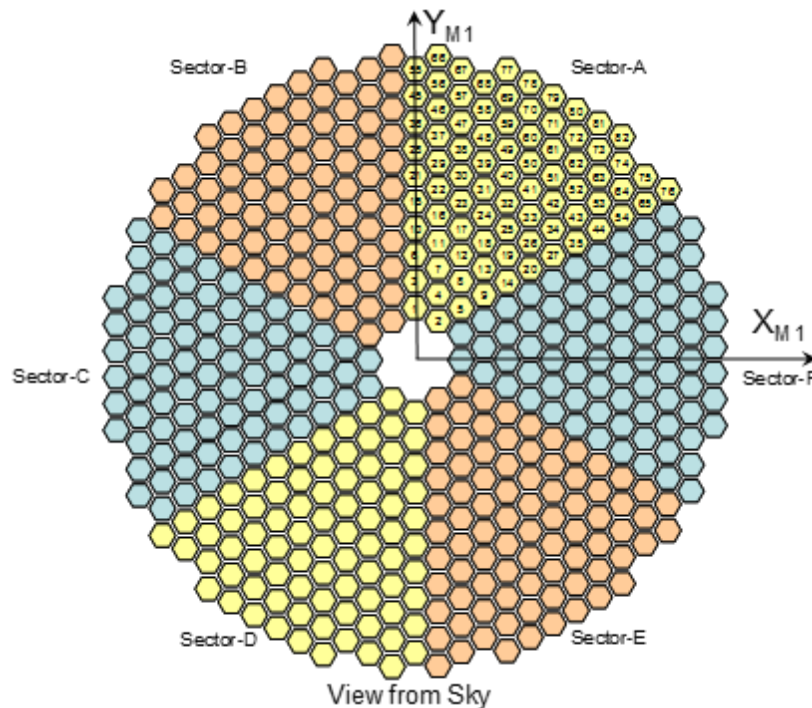


Figure 2-1. The global $M1$ Coordinate System for the TMT primary mirror and identification of the six Sectors and 82 Segment Types.

2.2.2 PSA Coordinate System

Definition: Each of the 82 Types of Segments is repeated every 60 degrees around the Z-axis. Figure 2-1 identifies the six sectors and each Type of Segment by its relative position in the primary mirror. The local coordinate system used in this document is based on the orientation of the SSA, and coordinates in this system are designated by the subscript PSA (Primary Segment Assembly), for example: X_{PSA} . The origin of the PSA Coordinate System resides on the Optical Surface at the nominal center of the Segment. The Z_{PSA} -axis is the normal to the Optical Surface at the origin. The X_{PSA} -axis and Y_{PSA} -axis are perpendicular to the Z_{PSA} -axis in a right-handed coordinate system. The position of the origin and the orientation of the X_{PSA} - and Y_{PSA} -axes have been chosen to provide the best orientation of the Segment with respect to the support system for each Segment Type. The Segments and PSA coordinate system rotate by 60 degrees when segments are moved from one sector to a neighboring sector, etc.). As such, the projection of the X_{PSA} - and Y_{PSA} -axes will not, in most cases, coincide with the Segment vertices, and they will not be parallel to the X_{M1} - and Y_{M1} -axes. The mechanical dimensions of the Segment are defined with respect to the PSA Coordinate System. More details on the definition of the PSA Coordinate System are provided in Appendix A (Section 5.1).

2.3 DIMENSIONAL TOLERANCES

[SPE-M1.SEG.POL-1410] The finished PMA, depicted in Figure 2-2, shall meet all requirements specified in (AD6).

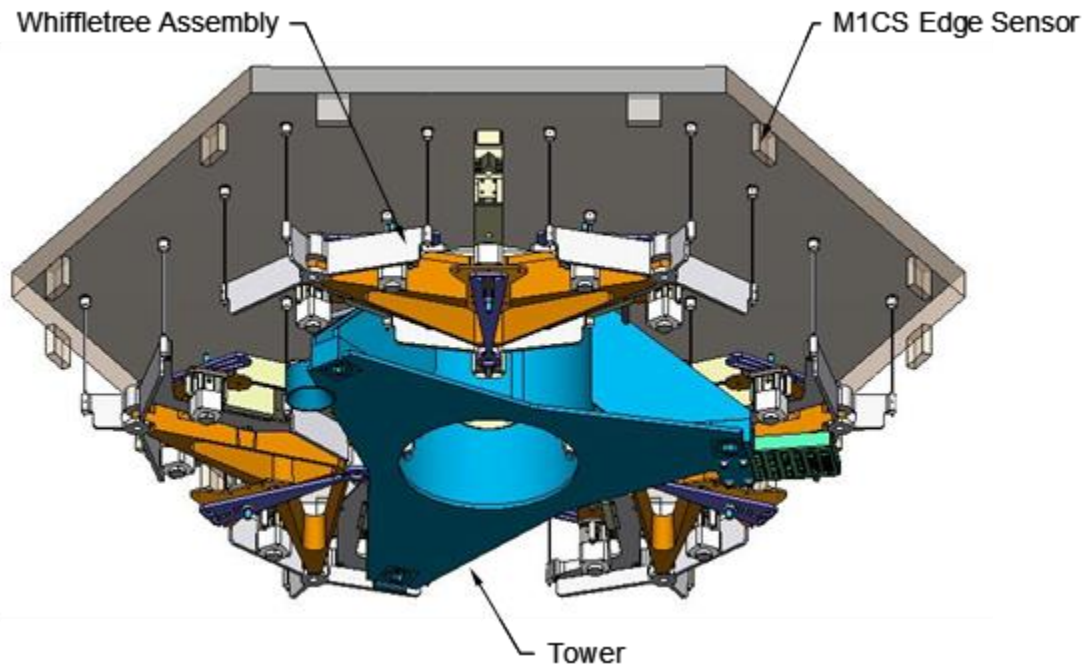


Figure 2-2. Polished Mirror Assembly with M1CS Edge Sensor simulators.

2.4 OPTICAL SURFACE

Definition: The Optical Surface is the entire polished concave surface of the Segment, extending to the bevels.

2.4.1 Surface Figure Accuracy

Discussion: Surface figure requirements must be met when both the measured aberrations and the measurement uncertainty are combined. The vendor will propose the method to measure the surface figure at all spatial frequencies and submit to TMT for approval. An example of an acceptable test method is described in section 2.3 of (RD5).

Definition: The TMT primary mirror is a hyperboloid. The expression for a conic surface of revolution is:

$$Z_{M1}(X_{M1}, Y_{M1}) = \{R - [R^2 - (K + 1)r^2]^{1/2}\} / (K + 1)$$

where K is the conic constant, R is the paraxial radius of curvature, and $r = (X_{M1}^2 + Y_{M1}^2)^{1/2}$. X_{M1} , Y_{M1} and Z_{M1} are in the M1 Coordinate System, as defined in Section 2.2.1.

[SPE-M1.SEG.POL-1925] For the TMT primary mirror, the conic constant $K = -1.00095348$ and the paraxial radius of curvature $R = 60.0$ meters. The conic constant and the paraxial radius of curvature shall be considered Basic Dimensions.

[SPE-M1.SEG.POL-1926] Segments shall have individual theoretical surface shapes described by Zernike terms as defined in (AD10). Zernike terms shall be treated as Basic Dimensions.

[SPE-M1.SEG.POL-1928] The surface figure errors of the PMA relative to the theoretical shape of each Optical Surface shall meet the following requirements using the vendor table specification (AD9)).

- PSSN shall be greater than 0.96 (as calculated by cell J8 of (AD9)).
- Adaptive Optics WFE shall be less than 15nm_{RMS} (as calculated by cell 08 of (AD9)).
- Maximum stroke of any warping harness actuator shall be less than 20% to correct low frequency errors (as calculated by cell T8 of (AD9)).

Discussion: Section 2.2 of (AD9) explains the computation method to be used by the vendor to calculate the above metrics for each segment.

[SPE-M1.SEG.POL-1929] The maximum allowable surface RMS shall be 5 nm over any 60 mm x 60 mm square with one edge aligned with the segment edge for all 6 edges. The RMS value is after tip/tilt removal over the 60 mm square aperture and measured down to spatial frequencies of 8 mm (TBR).

2.4.2 Surface Finish

[SPE-M1.SEG.POL-1930] The Optical Surface shall have a surface roughness of 20 Angstroms RMS or better.

Discussion: The roughness measurement shall capture all features with a spatial period between 20um and 800um. Roughness shall be calculated as the average of at least 10 samples spaced uniformly over the Optical Surface.

2.4.3 Scratch-Dig Specification

[SPE-M1.SEG.POL-1940] The scratch-dig specification for the Optical Surface is 60-40.

Discussion: The first number is the maximum width of a scratch in microns and the second number is the maximum diameter of digs in units of 0.01 mm, per MIL-PRF-13830B (RD2).

2.4.4 Segment Edge Figure

[SPE-M1.SEG.POL-1950] The Optical Surface region within the three lateral extent bands, 0-10 mm, 10-25 mm and 25-40 mm from the segment edge (excluding the bevel) shall each have surface figure errors relative to the theoretical surface shape of less than 10 nm RMS.

Discussion: The plans for measuring and verifying this requirement shall be submitted to TMT for approval.

2.4.5 Cracks

[SPE-M1.SEG.POL-2110] No visible cracks shall be allowed in the Segment. If a crack develops in a Segment surface, the crack shall be ground-out, leaving a depression that is approximately spherical. The depth of any such spherical depression shall be less than half the diameter of the sphere. A ground out spherical depression shall be considered to be a Chip as defined in this specification (Section 2.4.6).

2.4.6 Chips

[SPE-M1.SEG.POL-2115] No Chip shall exceed 10 mm in mean diameter after grinding. No more than three Chips are allowed on any Segment. No more than one Chip is allowed on the Optical Surface, and no more than one Chip is allowed on the Back Surface. No Chip is allowed to be within 10mm of any bonded support location or within 10mm from an edge sensor mounting location.

Discussion: A Chip is defined as a hollow depression in a surface of the Segment, usually formed where a flake has broken out of the mirror. All surfaces of a Chip must be ground out to remove sharp edges and cracks.

3. TESTING REQUIREMENTS

3.1 OPTICAL SURFACE TESTING CONFIGURATION

Discussion: The Polished Mirror Assembly (i.e. the assembly that must undergo final acceptance testing) is shown in Figure 2-2. Note that M1CS Edge Sensors will not be installed before the acceptance test, but must be simulated with dummy masses. In the telescope, the base of the SSA will be attached to a Subcell mounted on the primary mirror cell, with the piston, tip/tilt of the Segment controlled by three actuators that attach between the Subcell and the PMA.

[SPE-M1.SEG.POL-2018] The height of the Segment optical surface above the base plane of the SSA Tower Assembly during testing shall be as specified in (AD6).

Discussion: To control the height and orientation of the Segment in the Acceptance Optical Test, an Optical Testing Fixture shall take the place of the Subcell and actuators.

[SPE-M1.SEG.POL-2022] The Acceptance Optical Tests shall be performed with the Optical Surface facing upwards, i.e. with the Z_{PSA} -axis vertical. The warping harnesses shall be adjusted so that they do not exert any moments on the Segment.

[SPE-M1.SEG.POL-2024] Dummy masses shall be installed on the back of the segment at 12 points around the edge to simulate the mass distribution of the assembly in the telescope. The dummy masses are defined by TMT drawing M1S 910-00802 (RD6).

Discussion: In the telescope, the mass of the edge sensors will have a small but non-negligible effect on the segment figure.

[SPE-M1.SEG.POL-2026] In the Acceptance Optical Test, the Segment shall be positioned such that its figure is measured relative to the true position defined by the interface features on the Tower Assembly, within a tolerance of ± 50 microns in the X_{PSA} and Y_{PSA} directions and a tolerance of ± 250 micro-radians of rotation about the Z_{PSA} axis.

Discussion: The Segment shall be aligned in the optical test to ensure that the Segment will have the correct optical figure for its position in the primary mirror when it is mounted in the telescope. Alignment of the Segment in the telescope is accomplished by the registration features on the base of the Tower Assembly of the SSA. Any error or uncertainty in the segment position or rotation during the Acceptance Optical Test shall be considered when determining compliance to figure accuracy requirements.

[SPE-M1.SEG.POL-2028] The full-aperture test shall be performed with the Segment and its support system between 18 and 23 deg C and maintained between in thermal equilibrium within $\pm 1^\circ\text{C}$ of the nominal temperature during testing.

Discussion: The temperature during the test should be recorded and reported.

3.2 OPTICAL SURFACE TESTING ACCURACY

[SPE-M1.SEG.POL-2034] All tests and measurements used to verify compliance with the requirements of this specification shall be of sufficient accuracy to ensure the requirements have been met with a 1-sigma probability. This means that the measured values shall be sufficiently within the allowable range that, when measurement error is included, there is a 1-sigma probability that the parameter being measured is in compliance with the requirement.

[SPE-M1.SEG.POL-2038] No amount of focus term may be subtracted from the interferometry data. Only tip tilt and piston may be removed.

Discussion: Validated test-set-specific back outs may be permissible pending TMT written prior approval.

4. INSPECTION REPORT

[SPE-M1.SEG.POL-8000] A final end item data package shall be delivered with each Polished Mirror Assembly. A paper copy shall be packed with the PMA, and an electronic version (contained in a single data file) shall be provided to TMT. Interferometric data shall be in surface error data files that can be read and analyzed in Matlab. Scanned images of plots shall not be acceptable. This data package shall include, at a minimum, the following information.

- Segment Identification
- Segment Type Number
- Segment Serial Number
- Blank Serial Number
- Optical Metrology Data
- Full-aperture metrology interferometer data files
- Sub-aperture metrology data files (e.g. interferometer data files of edge measurements)
- Surface Roughness measurement results
- Error analysis supporting a 90% confidence level that accepted part meets all specifications, considering all potential error sources
- A dimensional inspection report, including the following information
- Compliance with all dimensions and drawing requirements of (AD6)
- Compliance with Scratch/Dig specifications
- Compliance with Crack and Chip limitations
 - Location of any Chips

5. APPENDICES

5.1 APPENDIX A. DESCRIPTION AND INSTRUCTIONS FOR USING M1S SEGMENTATION DATABASE (AD10)

5.1.1 Primary Mirror Coordinate Systems and Associated Notation

Two sets of coordinate axis systems are used to describe the primary mirror:

- a global system, designated as the *M1 Coordinate System*, and denoted by the subscript $_M1$;
- a series of local systems designated as *PSA Coordinate Systems*, and denoted by the subscript $_PSA$. There are as many PSA systems as there are segments.

All axis systems use the normal conventions for right hand, orthogonal Cartesian systems; in particular, positive rotations are always in the conventional, right hand direction relative to the coordinate axes.

Each one of the 492 segments in the array has its own, unique, local coordinate system $(xyz)_i$. Because of the 6-fold symmetry of the array, however, local systems (and segment geometry) need only be defined once for each of the 82 segment Types. When going from one sector to another, the segments, support systems, and local coordinate systems, all rotate together by multiples of 60° about the global Z axis.

The local system is defined in such a way that the location of the segment support system (axial support points, etc.) relative to the local axis system is identical for all segments.

The origins of the coordinate systems are denoted as $O_{PSA,i}$ and O_{M1} . Unit vectors along the positive direction of the coordinate axes are denoted $\bar{I}_{X,M1}$, $\bar{I}_{Y,M1}$, and $\bar{I}_{Z,M1}$ for the global system, and $\bar{I}_{X,PSAi}$, $\bar{I}_{Y,PSAi}$, and $\bar{I}_{Z,PSAi}$ for the local system attached to segment $\#i$.

5.1.2 Segment and Vertex Numbering

Segments within sector A are numbered from 1 to 82 as shown in Figure 2-1 of the main document. The same numbers are used to distinguish segment Types. Within each segment, vertices are numbered counter-clockwise from 1 to 6, starting with the vertex closest to the positive local X_{PSA} axis, as shown on the drawings, (RD2) and (AD6).

5.1.3 Detailed Definition of the Local Segment Reference System $(XYZ)_{PSA}$

The local PSA axis system results from a series of mathematical operations that are designed and adjusted to maximize the expected performance of the segment support system, by minimizing the deviation between actual segment outlines and a nominal, regular hexagon used in the design of the support system.

Because of this, there is no simple definition of the local frames $(XYZ)_{PSA,i}$.

Section 2 of the segmentation database, (AD10) contains definitions of the local PSA coordinate systems (in sector A), expressed as follows:

- the coordinates of the center of the segment (O_{PSA}), expressed in the M1 system
- the components, expressed in the M1 system, of unit vectors $\bar{I}_{X,PSA}$, $\bar{I}_{Y,PSA}$, $\bar{I}_{Z,PSA}$ along the coordinate axes X_{PSA} , Y_{PSA} , Z_{PSA} .

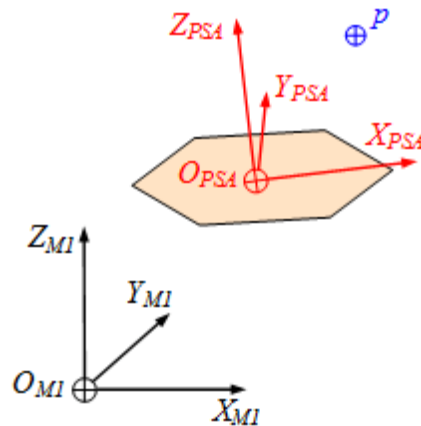


Figure 5-1. Coordinate transformation between the M1 system and a PSA system.

Given the coordinates of any point p , expressed in the local PSA axis system (p_{PSA}) (Figure 5-1), the coordinates of the same point, expressed in the M1 system (p_{M1}) are given by

$$p_{M1} = O_{PSA_{M1}} + \begin{bmatrix} \bar{i}_{X,PSA_{M1}} & \bar{i}_{Y,PSA_{M1}} & \bar{i}_{Z,PSA_{M1}} \end{bmatrix} p_{PSA} = O_{PSA_{M1}} + R_{M1}^{PSA} p_{PSA}, \quad (1)$$

where $O_{PSA_{M1}}$ is a column vector (3x1) containing the coordinate of the origin of the PSA system, expressed in the M1 system, and $R_{M1}^{PSA} = \begin{bmatrix} \bar{i}_{X,PSA_{M1}} & \bar{i}_{Y,PSA_{M1}} & \bar{i}_{Z,PSA_{M1}} \end{bmatrix}$ is the rotation matrix (3x3) from the M1 system to the PSA system, formed as the juxtaposition of the three column vectors ($\bar{i}_{X,PSA_{M1}}$, $\bar{i}_{Y,PSA_{M1}}$, and $\bar{i}_{Z,PSA_{M1}}$) containing the components of the unit vectors of the PSA system, expressed in the M1 system.

Note that since R_{M1}^{PSA} is ortho-normal, the inverse transformation is

$$p_{PSA} = (R_{M1}^{PSA})^T (p_{M1} - O_{PSA_{M1}}),$$

where the subscript T denotes the transpose of the rotation matrix.